Digital PLL Design and Analysis Topics

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Eric Jacobsen

Agenda

- **Introduction and Agenda**
- **Analog PLL Analysis**
- **Z-transformation**
- **Secret Tricks**
- **Summary**

Analog PLL Analysis

- For this presentation the Sacred Text of Gardner[1] will provide the foundation of the analysis
- A second-order, PI (proportional-integral) loop topology will be assumed
  - Extensions to other topologies are not complicated

12 Loop Fundamentals

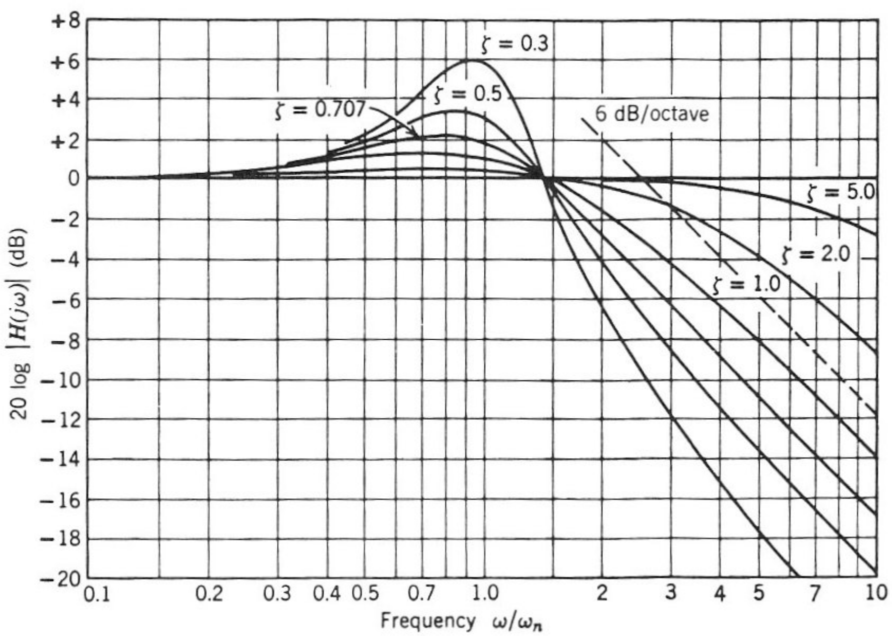
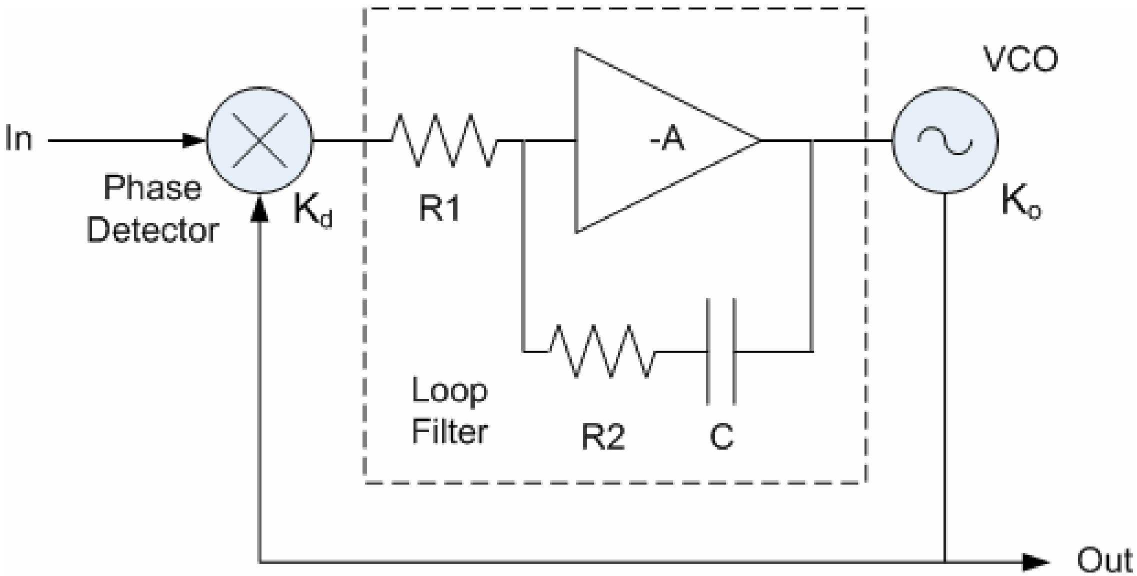


Figure 2.3 Frequency response of a high-gain second-order loop.

Figure 2.3 from Gardner shows the frequency response of a 2nd order PLL as characterized by the Natural Frequency, ω_n, and Damping Ratio, ζ.

1. Floyd M. Gardner, *Phaselock Techniques*, John Wiley & Sons, 1979

Analog PLL Analysis - 2



Using Gardner's analysis for an active 2^{nd} order loop:

$$\omega_n = \sqrt{\frac{K_o \cdot K_d}{\tau_1}}$$

$\tau_1 = R_1C$, $\tau_2 = R_2C$

ω_n = Natural Frequency

$$\zeta = \frac{\tau_2}{2} \cdot \sqrt{\frac{K_o \cdot K_d}{\tau_1}}$$

ζ = Damping Ratio

$$F(s) = \frac{s \cdot \tau_2 + 1}{s \cdot \tau_1}$$

S-domain transfer function

The basic loop components are the Phase Detector, Loop Filter, and VCO.

The circuit locks an output signal which can be some multiple or sub-multiple (factor) of the input signal frequency, to the input signal phase.

The Phase Detector and VCO have gain characteristics that are described by K_o and K_d.

Z-transformation

Simplifying the s-domain representation of the Loop Filter:

$$F(s) = \frac{\tau_2}{\tau_1} + \frac{1}{s\cdot\tau_1} = a + \frac{b}{s}$$ where $$a = \frac{\tau_2}{\tau_1}$$ and $$b = \frac{1}{\tau_1}$$

A translation to z-domain can then be done using various techniques. We elect a step-invariant method using:

$$H(z) = \frac{z-1}{z}\cdot Z\left[L^{-1}\cdot\left(\frac{F(s)}{s}\right)\right]$$ which yields:

$$H(z) = \frac{z-1}{z}\cdot Z\left[L^{-1}\cdot\left(\frac{a}{s} + \frac{b}{s^2}\right)\right] = \frac{z-1}{z}\cdot\left[\frac{a\cdot z}{z-1} + \frac{b\cdot T_s\cdot z}{(z-1)^2}\right] = a + \frac{b\cdot T_s\cdot z^{-1}}{1-z^{-1}} = \alpha + \frac{\beta\cdot z^{-1}}{1-z^{-1}}$$

The following relationships hold:

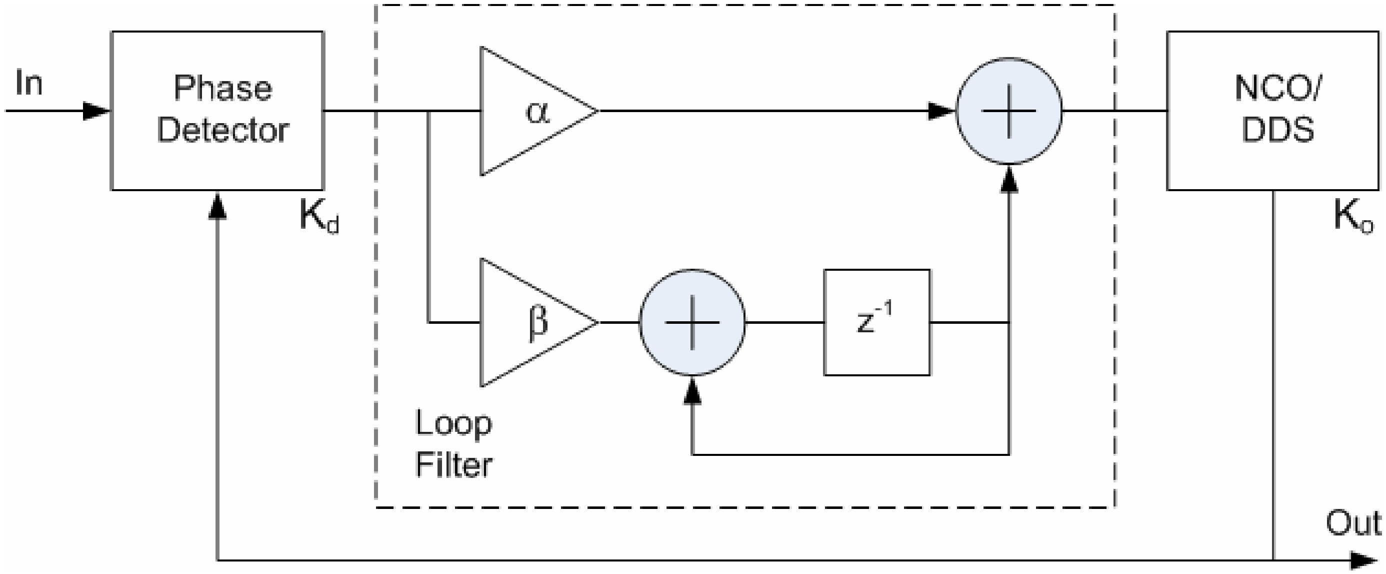
$$\alpha = \frac{\tau_2}{\tau_1} \qquad \beta = \frac{T_s}{\tau_1} = \frac{1}{\tau_1\cdot R_s} \qquad \omega_n = \sqrt{\frac{K_o\cdot K_d}{\tau_1}} = \sqrt{\beta\cdot R_s\cdot K_o\cdot K_d} = \sqrt{\beta\cdot R_s}\cdot\sqrt{K_o\cdot K_d}$$

$$\zeta = \frac{\tau_2}{2}\cdot\sqrt{\frac{K_o\cdot K_d}{\tau_1}} = \frac{\alpha}{2\cdot\beta\cdot R_s}\cdot\sqrt{\beta\cdot R_s}\cdot\sqrt{K_o\cdot K_d} = \frac{\alpha\cdot\sqrt{K_o\cdot K_d}}{2\cdot\sqrt{\beta\cdot R_s}}$$

$$\alpha = \frac{\zeta\cdot 2\cdot\sqrt{\beta\cdot R_s}}{\sqrt{K_o\cdot K_d}} = \frac{\zeta\cdot 2\cdot\sqrt{\frac{\omega_n^{\,2}}{R_s\cdot K_o\cdot K_d}\cdot R_s}}{\sqrt{K_o\cdot K_d}} = \frac{\zeta\cdot 2\cdot\omega_n}{K_o\cdot K_d} \qquad \beta = \frac{\omega_n^{\,2}}{R_s\cdot K_o\cdot K_d}$$

Z-Transformation - 2

The Z-transform solution leads to the Loop Filter topology shown here. α and β can be implemented as multiplies or restricted to powers-of-two integer shifts.



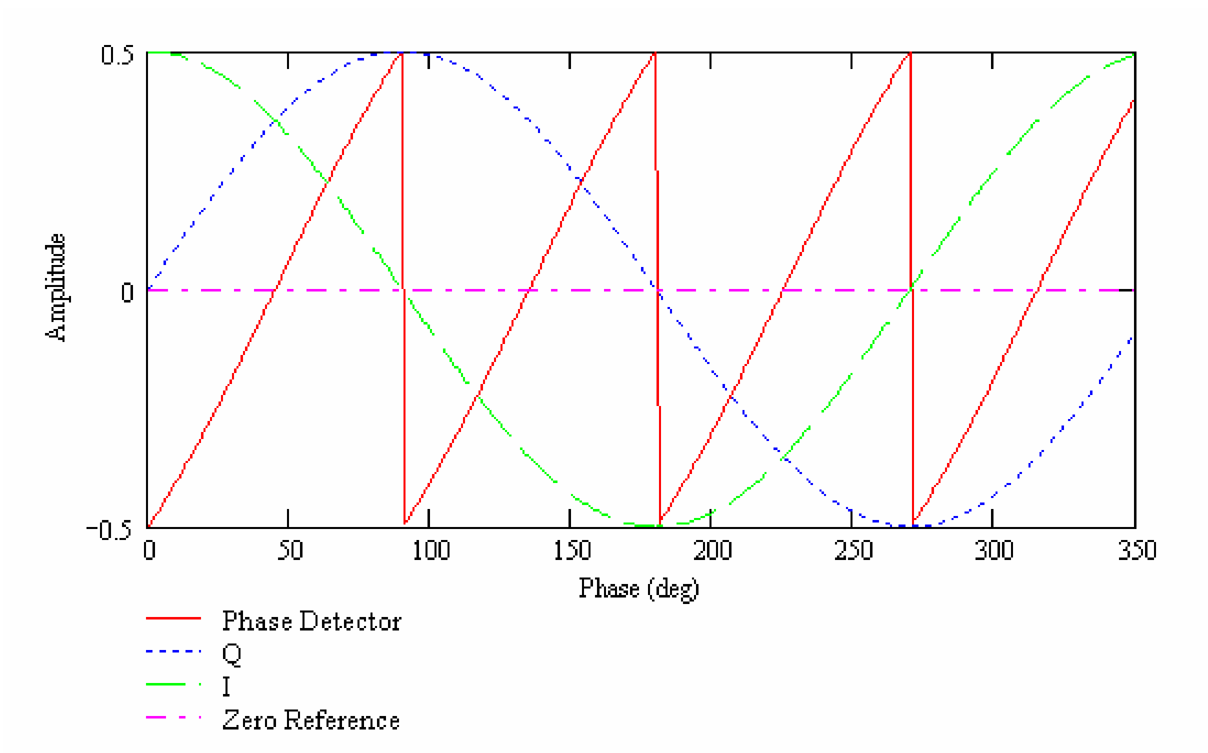
The input multiplier has been replaced with a suitable digital phase detector, and the VCO has been replaced with an NCO/DDS. The next step is analyzing these components and their characterizing gains K_o and K_d.

Secret Tricks

- Key elements of the analysis of the analog loop are the detector and VCO gains, K_o and K_d
  - The detector gain, K_d has units of Volts/Radian
  - The VCO gain, K_o has units of Radians/Second-Volt
- Both terms represent the slope of the linear region of operation of the device
- The product K_oK_d has units of 1/Second
- In digital systems Phase Detectors take many forms depending on the application
  - The detector gain can be analyzed graphically
- In digital systems the VCO is replaced by an NCO/DDS
  - The NCO gain can be analyzed graphically
- Selection of suitable units for K_o and K_d that provide accurate analysis is critical

Secret Tricks - 2

Shown below is the output of a QPSK phase detector, which has "lock points" at odd multiples of 45-degrees. The slope in the "lock" region for this signal amplitude is 1.0 per 90 degrees, or 2.0 per π radians. The key observation is that precision-related units can be used for the amplitude component, in this case, an MSB of precision.

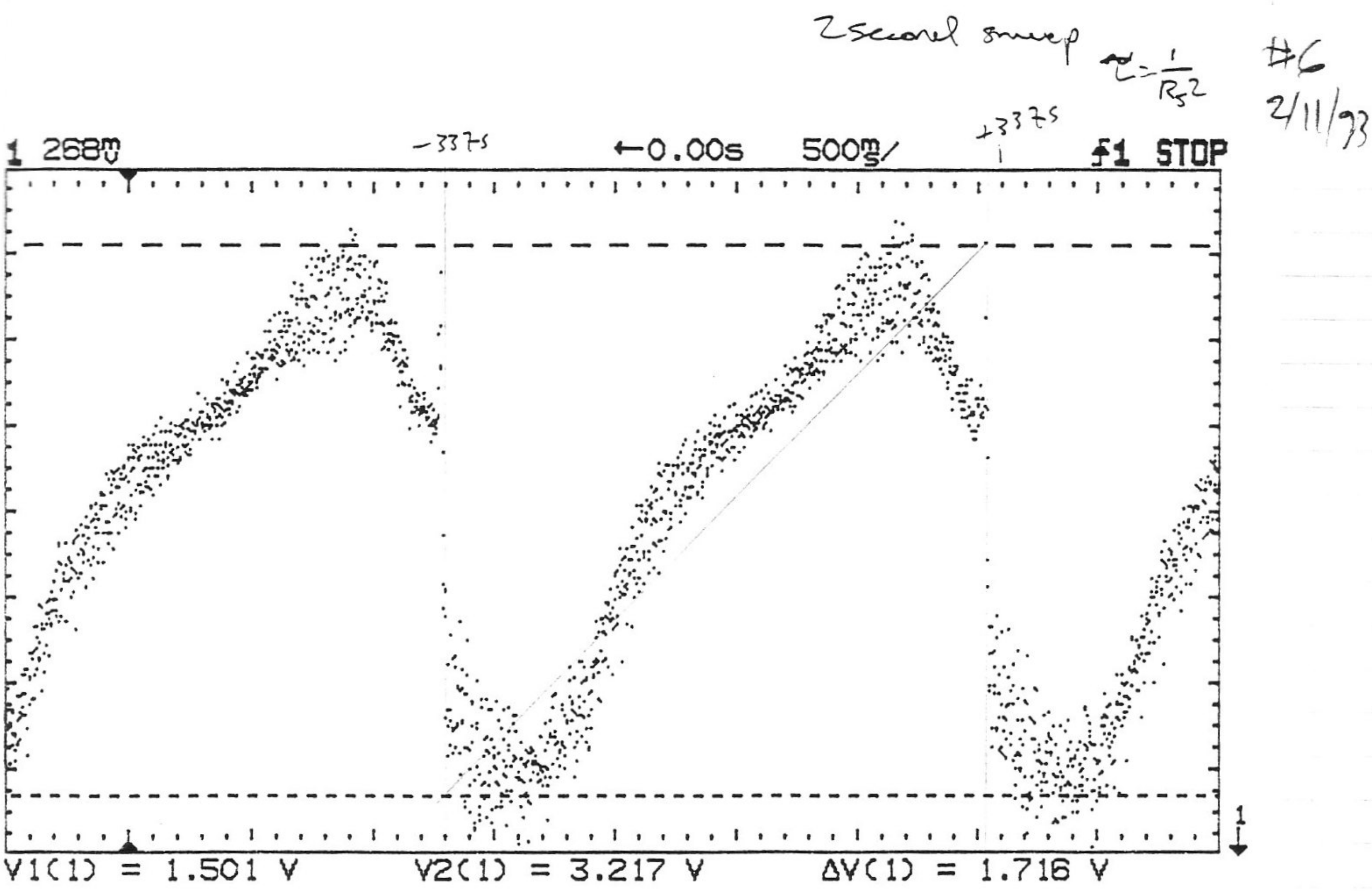


So for this detector, where the maximum output value is 1.0, $K_d = 1/\pi$ MSB/rad. i.e., a set MSB = 1 radian.

This can be easily achieved in a system implemented with a fractional, fixed-point format.

Secret Tricks - 3

Here a practical Frequency Discriminator implementation is exercised in noise in order to verify the implemented K_d .



Secret Tricks - 4

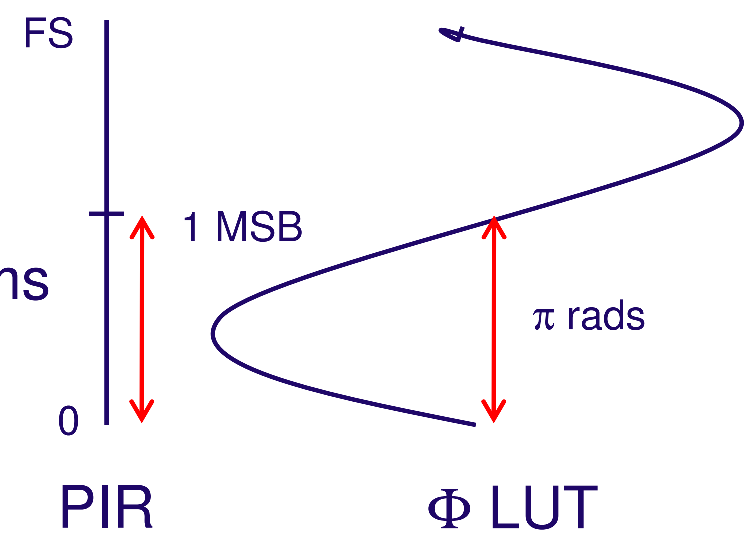
In order for the K_oK_d product to have units of 1/s when K_d has units of MSB/rad, the NCO gain, K_o, must have units of rad/s-MSB. When only the MSB of the Phase Increment Register (PIR) is set, one-half of the phase LUT is stepped through per NCO clock cycle. If the NCO phase LUT has one full sinusoidal cycle across the entire address range, then half of that, or one MSB, is traversed per NCO clock cycle.

So K_o is then

$K_o = \pi f_{NCOclk}$ rad/s-MSB

or, if one prefers to think in terms of phase per NCO clock cycle

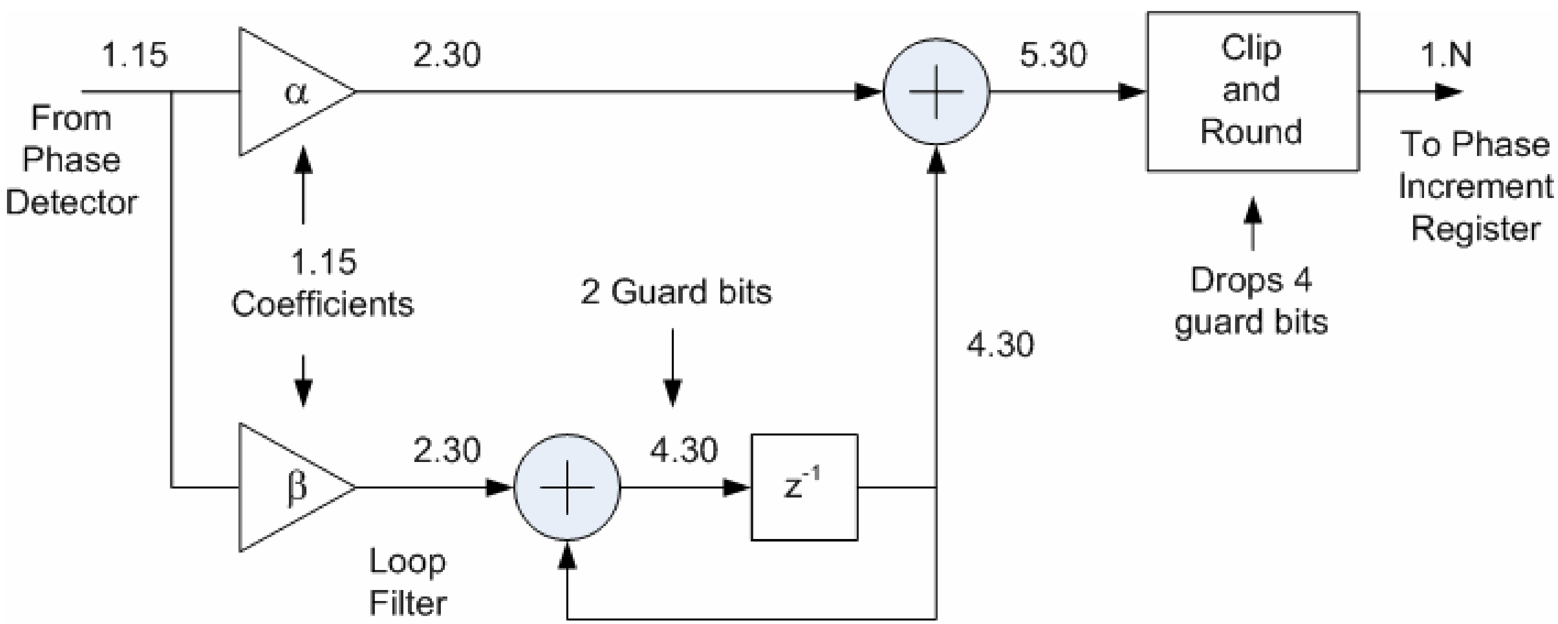
$K_o = \pi / T_{NCOclk}$ rad/s-MSB



Secret Tricks - 5

The key for consistency is that the 'MSB' in the Phase Detector must align, wrt the numerical precision, with the MSB of the NCO PIR input. This means accounting for precision and bit alignment through the Loop Filter.

An arbitrary example is shown below, with a clip and round stage prior to the NCO PIR to restore MSB alignment. *The precision and binary point location is indicated at each step.*



Secret Tricks - 6

- The unit used for the amplitude value of K_o and K_d is arbitrary, but must be kept consistent through the signal flow (e.g., MSB)
  - The K_oK_d product must have units of 1/s
  - ‘MSB’ is convenient because precision can be changed in the implementation without affecting the analysis or behavior
- Other Z-transformation techniques can be used with subtle tradeoffs
  - e.g., Bilinear, Impulse invariant
- DPLLs are reasonably insensitive to design errors
  - SNR affects things like K_d, and factors-of-two errors are not catastrophic, which can be seen in the analysis
  - Actually kind of hard to screw up if the basics are in place
- Measured implementations reveal high correlation between analysis and results – i.e., these methodologies appear accurate and reliable

Summary

- Z-transformation of the PLL Loop Filter yields formal expressions for loop control
  - ω_n, ζ, fully controllable
- The Phase Detector and NCO gains can be determined graphically or analytically
- Careful management of arithmetic precision (e.g., binary point alignment) is necessary for accurate analysis wrt K_o and K_d